# TEACHING MATHEMATICS: A COMPARISON OF CUISENAIRE RODS AND TRADITIONAL MATERIALS

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# TEACHING MATHEMATICS: A COMPARISON OF CUISENAIRE RODS AND TRADITIONAL MATERIALS

A Thesis
Presented to
the Faculty of
the Department of Childhood Education
Appalachian State University

In Partial Fulfillment
of the Requirements for the Degree
Master of Arts

by
Patricia L. Fagan
August 1972

# ABSTRACT

In this study, two groups of second grade children were taught mathematics using traditional mathematics materials and Cuisenaire rods for periods of fifteen weeks. The increases in arithmetic achievement scores were analyzed by the use of chi-square. The .05 level of confidence was used as a basis for rejection of the null hypotheses. Null hypothesis one, that when instructed with Cuisenaire rods versus traditional mathematics materials, there is no significant difference in the total arithmetic achievement scores of Group A versus Group B, was accepted. Null hypothesis two, that when instructed with Cuisenaire rods versus traditional mathematics materials, there is no significant difference in the Concepts subtest scores of Group A versus Group B, was rejected. Null hypothesis three, that when instructed with Cuisenaire rods versus traditional mathematics materials, there is no significant difference in the Reasoning subtest scores of Group A versus Group B, was accepted. Null hypothesis four, that when instructed with Cuisenaire rods versus traditional mathematics materials, there is no significant difference in the Computation subtest scores of Group A versus Group B, was accepted.

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## Chapter 1

# INTRODUCTION

More advances in mathematics have been made during the twentieth century than in all previous centuries in the history of the world (Marks, Purdy, Kinney, 1965). This fact alone necessitates reexamination of our schools' mathematics curriculum and teaching methods. Group instruction has become inadequate to stimulate and guide the learning of children whose backgrounds, interests, habits and practices are markedly different from the typical ones for whom the instructional program was originally designed (Gartner, Kohler, Riessman, 1971). Elementary mathematics programs need to be revised, enriched, and their emphasis readjusted. Today the demand for complex computational skills is limited. It has been replaced by the need to understand basic ideas, to discover important mathematical relationships, and to apply mathematical reasoning to new situations. This shift in the goals of mathematics education has resulted in a variety of new programs and materials designed to achieve these goals (Marks, Purdy, Kinney, 1965).

Among the new materials developed to promote discovery and understanding of mathematics concepts are some wooden sticks, known as Cuisenaire rods. These rods can be used at all grade levels, in conjunction with any textbook and within the framework of any mathematics curriculum (Davidson, 1969). The purpose of this study is to ascertain the effectiveness of Cuisenaire rods as compared to the effectiveness of traditional materials in the teaching of mathematics.

## The Need for this Study

Much has been said about a need for vast change in mathematics materials (Marks, Purdy, Kinney, 1965, and Gartner, Kohler, Riessman, 1971). The children of today, who are used to televisions, radios, record players, tape cassettes, photographs, slides, and movies, are not stimulated to learn by the use of pencils and papers (Flynn, 1972). Therefore, schools need to expand their programs to meet the needs of today's children by using stimulating, challenging materials.

The period starting with the mid 1950's has been characterized by a great deal of experimentation aimed at producing more effective teaching materials and methods. Attention has been focused on the improvement of instructional materials and procedures (Marks, Purdy, Kinney, 1965). But how effective have the new techniques and materials been in teaching mathematics? Many teachers believe that their pupils are more enthusiastic about learning arithmetic. They report that pupils enjoy discovering rules, thinking to solve problems, and seem to have a better understanding of mathematical concepts (Marks, Purdy, Kinney, 1965).

While the observations and evaluations of teachers seem to indicate that new mathematics materials are effective in teaching arithmetic, there is a need to substantiate this claim. This study is an attempt to investigate the effectiveness of one of the new mathematical materials: Cuisenaire rods.

## Statement of the Problem

The purpose of this study is to determine the effectiveness of using Cuisenaire rods in teaching mathematics to young children. More specifically the study is concerned with the following questions:

1. What is the effect on the total arithmetic scores of young children when Cuisenaire rods are used to teach mathematics as compared to the effect on the total arithmetic scores when traditional materials are used?

2. What is the effect on the Concepts subtest scores of young children when Cuisenaire rods are used to teach mathematics as compared to the effect on the Concepts subtest scores when traditional materials are used?

3. What is the effect on the Reasoning subtest scores of young children when Cuisenaire rods are used to teach mathematics as compared to the effect on the Reasoning subtest scores when traditional materials are used?

4. What is the difference between the effect on the Computation subtest scores of young children when Cuisenaire rods are used to teach mathematics as compared to the effect on the Computation subtest scores when traditional materials are used?

## Definition of Terms

Cuisenaire rods are wooden sticks which have a base of one square centimeter, and lengths varying from one centimeter to ten centimeters. They were developed by Georges Cuisenaire, a Belgian, in 1952 (Kunz, 1965). Rods of different lengths are color coded to enable the child to express a wide range of mathematical relationships. There are ten different lengths of rods, each having its own specific color. The rods can be used to solve addition, subtraction, multiplication, and division equations.

The Cuisenaire materials included the colored rods, the Cuisenaire workbook, Opening Doors in Mathematics, Book I, Part A (Genise and Kunz, 1971), and the Student Activity Cards for Cuisenaire Rods (Galton, Fair, and Davidson, 1971). The Cuisenaire method included teaching activities taken from Using the Cuisenaire Rods (Davidson, 1969), Mathematical Awareness (Trivett, 1962), Modern Mathematics Made Meaningful (Kunz, 1965), Mathematics with Numbers in Color (Gattegno, 1966), and the teacher's edition of Opening Doors in Mathematics (Genise and Kunz, 1971).

The traditional materials included the second grade mathematics workbook, Greater Cleveland Mathematics Program (Educational Research Council, 1968). The traditional method and activities were taken from the teacher's guide for the Greater Cleveland Mathematics Program. The traditional approach used whole class explanations and lecture-type instruction, whereas the Cuisenaire approach relied heavily on individual and small group instruction.

The children used in the samples included all pupils attending the school in the second grade at the time of testing. The school is an independent school located in Jacksonville, Florida.

## Delimitations of the Study

This study describes second grade pupils in terms of achievement scores using the Science Research Associates Achievement Series, Arithmetic 2-4, Form C and D (Thorpe, Lefever, Naslund, 1963, a, b.). The independent variables will not include teacher enthusiasm for a particular method, teacher competence in a particular method, or the mental and emotional state of the child, all of which do influence teaching-learning situations (Best, 1959). This study is an attempt to measure the significance of the differences in arithmetic achievement scores which are discovered through statistical analysis.

The findings of this study may be descriptive of second grade pupils in other schools; however, sample procedures prohibit generalization of the findings to other than the school from which the data were obtained.

## Chapter 2

## REVIEW OF LITERATURE

Much has been written concerning the discovery approach to learning and the use of concrete, manipulative models with young children. Hall (1965) undertook a study of discovery teaching techniques and materials to ascertain the effectiveness of such methods as measured by increased learning. After his study of several years, Hall reported that his results were

> so overwhelmingly favorable that there is considerable bias in favor of such methods, at least for a very wide range of topics most of which are commonly taught by the traditional approach. ("Traditional" usually implies either the lecture type of presentation or the teacher or text-given example-assignment type.) /Hall, 1965, p. 17/.

He also stated several observed conclusions about discovery techniques: (1) subject matter is retained longer, without extra review, (2) mathematical techniques were used with greater proficiency, (3) creativity was increased, (4) mathematics was enjoyed more, (5) and, a greatly increased sense of autonomous power and individual worth was noted (Hall, 1965). Hall's observations were not the result of a research design, but were determined by informal comparisons with other students taught over a ten year period.

Lovell (1961) discussed the growth of basic mathematical concepts in children and summarized Piaget's beliefs

> . . . that mathematical concepts are not derived from the materials themselves, but from an appreciation of the significance of the operations performed with the materials. The concepts and the ability to manoeuvre /sic/ them in the mind, he considers, are built up from using concrete material, but are independent of the actual materials used /Lovell, 1961, p. 45/.

Lovell described Cuisenaire rods as materials from which mathematical concepts can be effectively learned. He indicated that the rods meet Piaget's criteria of enabling the child to appreciate the significance of his own actions through rearrangement of materials, while yielding mathematically valuable concepts which rely only in part upon visual perception and imagery.

Callahan and Jacobson (1967) studied the effectiveness of using concrete, manipulative materials in teaching arithmetic to mentally retarded children. After using Cuisenaire rods for nine weeks, the researchers reported several findings: (1) the children retained mathematical concepts after the removal of the rods, (2) the concrete material enhanced the learning of number facts, (3) inverse operations were more easily recognized with rods, (4) the rods provided the opportunity to teach more advanced mathematics, on an elementary level, than conventional materials, (5) the results achieved were definitely better than might have been achieved, in the same time, without the rods, and (6) that mentally retarded children certainly do make discoveries, and also can retain information.

The comparative success of the conventional program, the Cuisenaire program and the Dienes program was studied by Brownell (1968) in England and Scotland. Thirteen Scottish programs and thirty-two English programs were observed. The Dienes program uses an apparatus known as Multibase Arithmetical Blocks, which were developed by Z. P. Dienes. The Multibase Arithmetical Blocks consist of "units," "longs," "flats," and "blocks." They are used to introduce children to the principles governing number rotation by working in bases three, four, five, and six before working in base ten.

In the conventional program, instruction was based on counting, grouping and regrouping. The children manipulated discrete objects first, and then relied on imaginative manipulation of pictorial representations. Emphasis was on learning the basic number combinations.

In the Scottish study, Brownell found that Cuisenaire materials, in general, were much more effective than conventional materials in developing meaningful mathematical abstractions, even though the Cuisenaire subjects had about twenty per cent less teaching time than the Conventional subjects (Brownell, 1968). As a group, the Cuisenaire subjects were rated slightly lower for brightness by the teachers, Brownell found that the Cuisenaire subjects exhibited a much greater maturity in thought processes to find the answers for the number combinations and a greater ability to explain the mathematical rationale behind computations than the subjects of the Conventional program.

The Conventional program had the highest overall rating for effectiveness in promoting conceptual maturity in the English study. The Cuisenaire and Dienes programs were ranked about equal. This reversal of effectiveness between the Conventional and Cuisenaire programs in the Scottish and English studies was explained to be the result of teaching. In the English study, the Cuisenaire program was taught more effectively than the Conventional program. But in the Scottish study, the Conventional program was taught better than the Cuisenaire program (Brownell, 1968).

Thus, studies have shown that discovery teaching techniques and materials lengthen the retention of subject matter, result in greater proficiency in the use of mathematical concepts, and increase creativity and enjoyment of mathematics (Hall, 1965). Also, it has been found that the growth of basic mathematical concepts is stimulated by the use of concrete, manipulative materials (Lovell, 1961). Furthermore, experiments indicate that concrete materials facilitate the teaching of more advanced mathematics, on the elementary level, than would ordinarily be taught (Callahan and Jacobson, 1967).

## Chapter 3

## METHODS AND PROCEDURES

This experiment was pre-post test design. The pretest used to classify the sample was the SRA Achievement Series, Arithmetic 2-4, (Form C) (Thorpe, Lefever, and Naslund, 1963). The first posttest (Posttest I) used to measure achievement was the SRA Achievement Series, Arithmetic 2-4, (Form D) (Thorpe, Lefever, and Naslund, 1963). The second posttest (Posttest II) used to measure achievement was the SRA Achievement Series 2-4 (Form C), (Thorpe, Lefever, and Naslund, 1963). These instruments measure a child's ability to recognize number symbols. This includes his understanding of cardinal and ordinal numbers, time, money, easy combinations and a few comparisons of quantity (Buros, 1965).

### The Sample

The subjects in this study were all of the pupils in the second grade during the school year 1971-72 at a particular school in Jacksonville, Florida. The pupils were divided into two matched groups, Group A and Group B, by the teachers, according to I.Q. scores from the Otis-Lennon Intelligence Test. Each group consisted of 17 subjects and one teacher.

Group A was taught mathematics using Cuisenaire materials for fifteen weeks from September, 1971, to January, 1972. From January to May, 1972, Group A was taught with traditional materials.

Group B was taught mathematics for fifteen weeks from September to January using traditional materials. Cuisenaire materials were used by Group B, for fifteen weeks, from January to May, 1972. In September the pretest was given to both groups. Posttest I was administered in January and Posttest II was administered in May, 1972.

## Design for Statistical Analysis

The 2 X 2 contingency table was employed to analyze the data (Downie and Heath, 1970). The independent variable was the use of Cuisenaire rods to teach mathematics. The four dependent variables were the test scores from the SRA Achievement Series, Arithmetic 2-4: (1) Total, (2) Concepts subtest, (3) Reasoning subtest, and (4) Computation subtest. Analysis was made for each dependent variable to determine if significant differences existed. The data was subjected to the chi-square test (Downie and Heath, 1970, p. 201):

$$X^2 = \frac{N[(ad)-(bc)]^2}{(k)(l)(m)(n)}$$

The contingency table consisted of four cells; therefore, the degree of freedom was one. The minimum level of acceptance for chi-square was at the .05 level.

## The Null Hypotheses

### Null Hypothesis One

When instructed with Cuisenaire rods versus traditional mathematics materials, there is no significant difference in the total arithmetic achievement scores of Group A versus Group B.

### Null Hypothesis Two

When instructed with Cuisenaire rods versus traditional mathematics materials, there is no significant difference in the Concepts subtest scores of Group A versus Group B.

### Null Hypothesis Three

When instructed with Cuisenaire rods versus traditional mathematics materials, there is no significant difference in the total Reasoning subtest scores of Group A versus Group B.

### Null Hypothesis Four

When instructed with Cuisenaire rods versus traditional mathematics materials, there is no significant difference in the Computation subtest scores of Group A versus Group B.

## The Cuisenaire Method

The Cuisenaire approach to mathematics differs from the traditional approach, especially during the initial stages. A child's first encounter with the rods is through free play. This period of free exploration may last for several weeks. The importance of free exploration cannot be over-stressed (Genise and Kunz, 1971). It is during this play that the child asks questions of the rods and discovers his own answers. Many of the relationships learned through this play may not be verbalized by the child, but nonetheless the relationships are being discovered. During the free play sessions, the teacher quietly observes the children and their interactions with the rods. The teacher may stimulate discoveries by asking thought-provoking questions or by commenting on the constructions being built with the rods.

The next stage of work with the rods becomes a little more formal. Keeping in mind the philosophy of discovery, lessons for small group instruction are planned. The color names of the rods are used throughout the lessons. Some basic concepts concerning the rods are discovered during this stage:

1. Rods of the same color have the same length.
2. Rods of the same length have the same color.
3. The rods vary in length. Some rods are shorter than others and some rods are longer than others.

4. Rods can be identified by just feeling them, without seeing them.

5. A staircase is built if the rods are put in order from the shortest to the longest.

6. Sets of equivalent rods can be formed.

7. Sets of equivalent trains can be formed. (A train is built by placing two or more rods end to end.) (Gattegno, 1966).

The next stage of work introduces notation. However, the notation is with the letter names of the rods and not with numerals. The letter names are a result of the shortened color names. For example, the red rod row becomes the "R" rod.

At this point in working with the rods, the child already knows the relationships of the rods and he is aware of what rods patterns are equivalent. Using letter notation, the child can now record the relationships. To facilitate notation, new vocabulary words and symbols are informally introduced. These include the terms plus, minus, equals, and equation. Now the child can write down that the train formed by a white rod and a red rod is equivalent to a green rod more simply as W + R = G.

All addition concepts and facts (up to ten) are introduced at this stage. And subtraction is introduced simultaneously, as an inverse operation. After the initial instruction, using the letter names for the rods, the rods do not need to be used to solve problems. However, they are used to verify computation. After the child has mastered the operations with letter names, the numerical names may be introduced. The most commonly used unit of measure employs the white rod as one. Using this unit, the rods have numerical value from one through ten. Now the equation $W + R = G$ becomes $1 + 2 = 3$.

Addition and subtraction are only two of the operations that can be introduced with rods. Upon examining the table of contents in Using the Cuisenaire Rods (Davidson, 1969) one finds many other concepts listed:

- multiplication
- division
- place value
- odd and even numbers
- factors
- prime factorization
- least common multiple
- least common denominator
- fractions
  - rational numbers
  - reciprocals
  - fractional parts
  - addition and subtraction of fractions
  - division of whole numbers by fractions
  - division of fractions and mixed numbers by fractions
  - fractions as operators-multiplication
- ratio and proportion-quadrants
- measurement-perimeter, area, volume
- bar graphs
- frequency distribution-mean, median, mode
- signed numbers
- introduction to algebra
- two digit multipliers and mixed number multipliers
- division algorisms-whole numbers
- ordinal and cardinal numbers
- modular or "clock arithmetic"
- symmetry
- probability

Chapter 4

# RESULTS

The data were analyzed by the chi-square and the following results were obtained:

## Null Hypothesis One

When instructed with Cuisenaire rods versus traditional mathematics materials, there is no significant difference in the total arithmetic achievement scores of Group A versus Group B. The data analyzed by chi-square yielded a value of .01. This value was not found to be significant at the .05 level; therefore, the null hypothesis must be accepted.

## Null Hypothesis Two

When instructed with Cuisenaire rods versus traditional mathematics materials, there is no significant difference in the Concepts subtest scores of Group A versus Group B. The data analyzed by chi-square yielded a value of 29.27. This value was found to be significant beyond the .05 level; thus, the null hypothesis must be rejected.

Null Hypothesis Three

When instructed with Cuisenaire rods versus traditional mathematics materials, there is no significant difference in the Reasoning subtest scores of Group A versus Group B. The data analyzed by chi-square yielded a value of 1.03. This value was not found to be significant at the .05 level; therefore, the null hypothesis must be accepted.

Null Hypothesis Four

When instructed with Cuisenaire rods versus traditional mathematics materials, there is no significant difference in the Computation subtest scores of Group A versus Group B. The data analyzed by chi-square yielded a value of 1.17. This value was not found to be significant at the .05 level; therefore, the null hypothesis must be accepted.

Chapter 5

# DISCUSSION AND CONCLUSIONS

From the results of the data of this study it can be concluded that the only significant difference in the mathematics achievement scores of second grade children, when using traditional materials versus Cuisenaire materials, is in the Concepts subtest. The increase in the Concepts scores while using Cuisenaire rods points out that mathematical concepts are learned decidedly more effectively with rods. The fact that the Cuisenaire rods are colorful, concrete, manipulative materials designed to stimulate interest and discovery may account for the improved learning. The discovery approach encouraged by the use of rods is one of the most important aspects of the Cuisenaire material. As John Kunz (1965, p. 17), Educational Director of the Cuisenaire Company, reiterates, "It is certainly common knowledge that anyone, child or adult, will more readily learn and more permanently retain facts and ideas that he has worked out for himself, as against a series of words that are not made meaningful to him through his own experiences."

The time spent in discovery and free play while using rods may be the reason that, in the total arithmetic, Reasoning and Computation scores, there was no significant difference between the two materials. The group using the Cuisenaire rods spent a great deal of time in creative construction at all stages. The more formal lessons came only after free exploration. However, the group using traditional materials, without free play sessions, had more formal lessons and went at a faster pace. Perhaps periods of fifteen weeks were not long enough to account for the time spent in free exploration. If the same experiment were conducted with periods of thirty weeks, the lengthened time periods might alleviate the time disadvantage of the rod groups.

The tables in the appendix list the subtest scores of Group A and Group B. The increases are indicated in the last column. The Pretest scores were taken from the tests given in September. Posttest I refers to the testing in January, and Posttest II refers to the testing in May. The total increases of the test scores were used in the contingency tables.

The significant difference found in the Concepts subtests of Group A and Group B, demonstrating that Cuisenaire rods were more effective in teaching mathematical concepts than traditional materials, seems to be puzzling. The value of chi-square was found to be 29.27, which is far above the 3.841 value needed to be significant at the .05 level of confidence. Yet Group B had a greater increase in test scores after using traditional materials. For Group B the increase, after using traditional materials, was 22 points more than the increase exhibited after using rods. But Group A had an increase of 45 points more after using rods than after using traditional materials. Therefore, the increase after using rods was more than double the increase measured after the use of traditional materials.

There are several limitations inherent in this experiment. The limitations of the size of the sample studied affects the outcomes. The switch from traditional materials to Cuisenaire rods for Group B and the switch from rods to traditional materials for Group A, within one school year, may have had a confusing effect upon the children which influenced the test scores. The test used was quite comprehensive, but it is questionable whether or not it sufficiently tested the concepts taught with the rods (Buros, 1965). The children learned about fractions, multiplication, and division while using the rods. The test was designed with a minimum of emphasis placed upon these concepts (Buros, 1965).

Another limitation was the duration of the research. Fifteen weeks were adequate, yet longer periods of time would have produced more meaningful results (Myers, 1969).

## Recommendations for Further Research

Further investigation into the effectiveness of Cuisenaire rods over traditional materials could greatly benefit the teaching of mathematics. Longitudinal studies of the relationship between the use of rods and the mathematics achievement of children should be conducted to discern if achievement gains are lasting. Research in the area of teacher training would be appropriate to find out if teachers trained in the use of rods are more effective than untrained teachers using rods. The introduction of rods into the mathematics curriculum at different grade levels should be investigated to discover if there is a critical level at which rods are most effectively introduced and used. As more research is conducted concerning Cuisenaire rods, teachers will be able to more effectively help children experience mathematics.

APPENDIX

Table 1

Concepts Subtest Raw Scores of Group A after Using Cuisenaire Rods

| Student | Pretest | Posttest I | Increase |
|---|---|---|---|
| 1 | 36 | 34 | -2 |
| 2 | 29 | 29 | 0 |
| 3 | 26 | 35 | 9 |
| 4 | 25 | 32 | 7 |
| 5 | 23 | 27 | 4 |
| 6 | 22 | 19 | -3 |
| 7 | 20 | 19 | -1 |
| 8 | 18 | 26 | 8 |
| 9 | 17 | 25 | 8 |
| 10 | 17 | 23 | 6 |
| 11 | 16 | 14 | -2 |
| 12 | 15 | 19 | 4 |
| 13 | 14 | 25 | 11 |
| 14 | 14 | 14 | 0 |
| 15 | 14 | 14 | 0 |
| 16 | 12 | 21 | 9 |
| 17 | 12 | 13 | 1 |
| Total | 330 | 389 | 59[a] |
| Mean | 19.41 | 22.88 | 3.47 |

[a]Used in contingency table.

Table 2

Concepts Subtest Raw Scores of Group B after Using Traditional Materials

| Student | Pretest | Posttest I | Increase |
|---|---|---|---|
| 1 | 32 | 35 | 3 |
| 2 | 26 | 30 | 4 |
| 3 | 26 | 28 | 2 |
| 4 | 23 | 27 | 4 |
| 5 | 22 | 28 | 6 |
| 6 | 22 | 28 | 6 |
| 7 | 20 | 27 | 7 |
| 8 | 20 | 20 | 0 |
| 9 | 18 | 27 | 9 |
| 10 | 18 | 23 | 5 |
| 11 | 17 | 13 | -4 |
| 12 | 16 | 20 | 4 |
| 13 | 13 | 17 | 4 |
| 14 | 12 | 13 | 1 |
| 15 | 11 | 19 | 8 |
| 16 | 9 | 17 | 8 |
| 17 | 7 | 15 | 8 |
| Total | 312 | 387 | 75[a] |
| Mean | 18.35 | 22.76 | 4.41 |

[a]Used in contingency table.

Table 3

Reasoning Subtest Raw Scores of Group A after Using Cuisenaire Rods

| Student | Pretest | Posttest I | Increase |
|---|---|---|---|
| 1 | 24 | 22 | -2 |
| 2 | 20 | 21 | 1 |
| 3 | 17 | 24 | 7 |
| 4 | 16 | 23 | 7 |
| 5 | 15 | 19 | 4 |
| 6 | 15 | 16 | 1 |
| 7 | 15 | 13 | -2 |
| 8 | 14 | 13 | -1 |
| 9 | 12 | 17 | 5 |
| 10 | 11 | 11 | 0 |
| 11 | 11 | 6 | -5 |
| 12 | 10 | 11 | 1 |
| 13 | 9 | 11 | 2 |
| 14 | 8 | 6 | -2 |
| 15 | 8 | 7 | -1 |
| 16 | 7 | 8 | 1 |
| 17 | 2 | 8 | 6 |
| Total | 214 | 236 | 22[a] |
| Mean | 12.58 | 13.88 | 1.29 |

[a]Used in contingency table.

Table 4

Reasoning Subtest Raw Scores of Group B after Using Traditional Materials

| Student | Pretest | Posttest I | Increase |
|---|---|---|---|
| 1 | 19 | 22 | 3 |
| 2 | 16 | 23 | 7 |
| 3 | 16 | 16 | 0 |
| 4 | 15 | 20 | 5 |
| 5 | 14 | 18 | 4 |
| 6 | 14 | 16 | 2 |
| 7 | 14 | 16 | 2 |
| 8 | 14 | 14 | 0 |
| 9 | 14 | 13 | -1 |
| 10 | 13 | 15 | 2 |
| 11 | 12 | 17 | 5 |
| 12 | 12 | 15 | 3 |
| 13 | 10 | 11 | 1 |
| 14 | 10 | 9 | -1 |
| 15 | 9 | 16 | 7 |
| 16 | 9 | 11 | 2 |
| 17 | 8 | 10 | 2 |
| Total | 219 | 262 | 43[a] |
| Mean | 12.88 | 15.41 | 2.52 |

[a]Used in contingency table.

Table 5

Computation Subtest Raw Scores of Group A after Using Cuisenaire Rods

| Student | Pretest | Posttest I | Increase |
|---|---|---|---|
| 1 | 19 | 25 | 6 |
| 2 | 19 | 18 | -1 |
| 3 | 16 | 15 | -1 |
| 4 | 14 | 15 | 1 |
| 5 | 12 | 22 | 10 |
| 6 | 11 | 15 | 4 |
| 7 | 11 | 13 | 2 |
| 8 | 11 | 10 | -1 |
| 9 | 11 | 2 | -9 |
| 10 | 10 | 9 | -1 |
| 11 | 9 | 11 | 2 |
| 12 | 9 | 3 | -6 |
| 13 | 8 | 12 | 4 |
| 14 | 8 | 11 | 3 |
| 15 | 5 | 10 | 5 |
| 16 | 5 | 6 | 1 |
| 17 | 3 | 6 | 3 |
| Total | 181 | 205 | 22[a] |
| Mean | 10.65 | 12.06 | 1.29 |

[a]Used in contingency table.

Table 6

Computation Subtest Raw Scores of Group B after Using Traditional Materials

| Student | Pretest | Posttest I | Increase |
|---|---|---|---|
| 1 | 17 | 18 | 1 |
| 2 | 16 | 19 | 3 |
| 3 | 15 | 20 | 5 |
| 4 | 12 | 19 | 7 |
| 5 | 12 | 14 | 2 |
| 6 | 11 | 16 | 5 |
| 7 | 11 | 13 | 2 |
| 8 | 10 | 24 | 14 |
| 9 | 10 | 12 | 2 |
| 10 | 8 | 13 | 5 |
| 11 | 8 | 11 | 3 |
| 12 | 7 | 11 | 4 |
| 13 | 6 | 9 | 3 |
| 14 | 5 | 11 | 6 |
| 15 | 4 | 13 | 9 |
| 16 | 4 | 13 | 9 |
| 17 | 1 | 2 | 1 |
| Total | 157 | 238 | 81[a] |
| Mean | 9.24 | 14.00 | 4.76 |

[a]Used in contingency table.

Table 7

Total Arithmetic Raw Scores of Group A after Using Cuisenaire Rods

| Student | Pretest | Posttest I | Increase |
|---|---|---|---|
| 1 | 79 | 81 | 2 |
| 2 | 61 | 67 | 6 |
| 3 | 58 | 78 | 20 |
| 4 | 58 | 63 | 5 |
| 5 | 48 | 63 | 15 |
| 6 | 42 | 41 | -1 |
| 7 | 42 | 36 | -6 |
| 8 | 41 | 51 | 10 |
| 9 | 40 | 57 | 17 |
| 10 | 37 | 35 | -2 |
| 11 | 36 | 46 | 10 |
| 12 | 36 | 22 | -14 |
| 13 | 34 | 36 | 2 |
| 14 | 33 | 36 | 3 |
| 15 | 31 | 43 | 12 |
| 16 | 30 | 25 | -5 |
| 17 | 19 | 27 | 8 |
| Total | 725 | 807 | 82[a] |
| Mean | 42.65 | 47.47 | 4.82 |

[a]Used in contingency table.

Table 8

Total Arithmetic Raw Scores of Group B after Using Traditional Materials

| Student | Pretest | Posttest I | Increase |
|---|---|---|---|
| 1 | 63 | 74 | 11 |
| 2 | 59 | 64 | 5 |
| 3 | 51 | 68 | 17 |
| 4 | 49 | 66 | 17 |
| 5 | 48 | 58 | 10 |
| 6 | 48 | 58 | 10 |
| 7 | 46 | 66 | 20 |
| 8 | 44 | 52 | 8 |
| 9 | 42 | 55 | 13 |
| 10 | 41 | 49 | 8 |
| 11 | 34 | 42 | 8 |
| 12 | 32 | 45 | 13 |
| 13 | 30 | 37 | 7 |
| 14 | 28 | 36 | 8 |
| 15 | 26 | 45 | 19 |
| 16 | 24 | 29 | 5 |
| 17 | 23 | 42 | 19 |
| Total | 688 | 886 | 198[a] |
| Mean | 40.47 | 52.12 | 11.65 |

[a]Used in contingency table.

Table 9

Concepts Subtest Raw Scores of Group A after Using Traditional Materials

| Student | Posttest I | Posttest II | Increase |
|---|---|---|---|
| 1 | 35 | 35 | 0 |
| 2 | 34 | 35 | 1 |
| 3 | 32 | 24 | -8 |
| 4 | 29 | 34 | 5 |
| 5 | 27 | 31 | 4 |
| 6 | 26 | 28 | 2 |
| 7 | 25 | 26 | 1 |
| 8 | 25 | 17 | -8 |
| 9 | 23 | 27 | 4 |
| 10 | 21 | 19 | -2 |
| 11 | 19 | 22 | 3 |
| 12 | 19 | 22 | 3 |
| 13 | 19 | 15 | -4 |
| 14 | 14 | 19 | 5 |
| 15 | 14 | 18 | 4 |
| 16 | 14 | 14 | 0 |
| 17 | 13 | 17 | 4 |
| Total | 389 | 403 | 14[a] |
| Mean | 22.88 | 23.71 | 0.82 |

[a]Used in contingency table.

Table 10

Concepts Subtest Raw Scores in Group B after Using Cuisenaire Rods

| Student | Posttest I | Posttest II | Increase |
|---|---|---|---|
| 1 | 35 | 36 | 1 |
| 2 | 30 | 30 | 0 |
| 3 | 28 | 34 | 6 |
| 4 | 28 | 32 | 4 |
| 5 | 28 | 30 | 2 |
| 6 | 27 | 35 | 8 |
| 7 | 27 | 30 | 3 |
| 8 | 27 | 29 | 2 |
| 9 | 23 | 27 | 4 |
| 10 | 20 | 22 | 2 |
| 11 | 20 | 21 | 1 |
| 12 | 19 | 21 | 2 |
| 13 | 17 | 28 | 11 |
| 14 | 17 | 11 | -6 |
| 15 | 15 | 21 | 6 |
| 16 | 13 | 17 | 4 |
| 17 | 13 | 16 | 3 |
| Total | 387 | 440 | 53[a] |
| Mean | 22.76 | 25.88 | 3.11 |

[a]Used in contingency table.

Table 11

Reasoning Subtest Raw Scores of Group A after Using Traditional Materials

| Student | Posttest I | Posttest II | Increase |
|---|---|---|---|
| 1 | 24 | 24 | 0 |
| 2 | 23 | 24 | 1 |
| 3 | 22 | 24 | 2 |
| 4 | 21 | 22 | 1 |
| 5 | 19 | 19 | 0 |
| 6 | 17 | 21 | 4 |
| 7 | 16 | 16 | 0 |
| 8 | 13 | 20 | 7 |
| 9 | 13 | 17 | 4 |
| 10 | 11 | 18 | 7 |
| 11 | 11 | 15 | 4 |
| 12 | 11 | 14 | 3 |
| 13 | 8 | 17 | 9 |
| 14 | 8 | 13 | 5 |
| 15 | 7 | 12 | 5 |
| 16 | 6 | 12 | 6 |
| 17 | 6 | 10 | 4 |
| Total | 236 | 298 | 62[a] |
| Mean | 13.88 | 17.52 | 3.65 |

[a]Used in contingency table.

Table 12

Reasoning Subtest Raw Scores of Group B after Using Cuisenaire Rods

| Student | Posttest I | Posttest II | Increase |
|---|---|---|---|
| 1 | 23 | 22 | -1 |
| 2 | 22 | 22 | 0 |
| 3 | 20 | 23 | 3 |
| 4 | 18 | 15 | -3 |
| 5 | 17 | 14 | -3 |
| 6 | 16 | 21 | 5 |
| 7 | 16 | 18 | 2 |
| 8 | 16 | 18 | 2 |
| 9 | 16 | 16 | 0 |
| 10 | 15 | 14 | -1 |
| 11 | 15 | 14 | -1 |
| 12 | 14 | 19 | 5 |
| 13 | 13 | 18 | 5 |
| 14 | 11 | 13 | 2 |
| 15 | 11 | 10 | -1 |
| 16 | 10 | 14 | 4 |
| 17 | 9 | 13 | 4 |
| Total | 262 | 284 | 22[a] |
| Mean | 15.41 | 16.71 | 1.29 |

[a]Used in contingency table.

Table 13

Computation Subtest Raw Scores of Group A after Using Traditional Materials

| Student | Posttest I | Posttest II | Increase |
|---|---|---|---|
| 1 | 25 | 27 | 2 |
| 2 | 22 | 24 | 2 |
| 3 | 18 | 20 | 2 |
| 4 | 15 | 24 | 9 |
| 5 | 15 | 23 | 8 |
| 6 | 15 | 12 | -3 |
| 7 | 13 | 13 | 0 |
| 8 | 12 | 24 | 12 |
| 9 | 11 | 12 | 1 |
| 10 | 11 | 3 | -8 |
| 11 | 10 | 15 | 5 |
| 12 | 10 | 8 | -2 |
| 13 | 9 | 6 | -3 |
| 14 | 6 | 16 | 10 |
| 15 | 6 | 8 | 2 |
| 16 | 3 | 6 | 3 |
| 17 | 2 | 12 | 10 |
| Total | 205 | 279 | 50[a] |
| Mean | 12.06 | 16.41 | 2.94 |

[a]Used in contingency table.

Table 14

Computation Subtest Raw Scores of Group B after Using Cuisenaire Rods

| Student | Posttest I | Posttest II | Increase |
|---|---|---|---|
| 1 | 24 | 25 | 1 |
| 2 | 20 | 22 | 2 |
| 3 | 19 | 20 | 1 |
| 4 | 19 | 20 | 1 |
| 5 | 18 | 21 | 3 |
| 6 | 16 | 15 | -1 |
| 7 | 14 | 25 | 11 |
| 8 | 13 | 18 | 5 |
| 9 | 13 | 18 | 5 |
| 10 | 13 | 13 | 0 |
| 11 | 13 | 12 | -1 |
| 12 | 12 | 16 | 4 |
| 13 | 11 | 14 | 3 |
| 14 | 11 | 13 | 2 |
| 15 | 11 | 12 | 1 |
| 16 | 9 | 22 | 13 |
| 17 | 2 | 2 | 0 |
| Total | 238 | 288 | 50[a] |
| Mean | 14.00 | 16.94 | 2.94 |

[a]Used in contingency table.

Table 15

Total Arithmetic Raw Scores of Group A after Using Traditional Materials

| Student | Posttest I | Posttest II | Increase |
|---|---|---|---|
| 1 | 81 | 86 | 5 |
| 2 | 78 | 81 | 3 |
| 3 | 67 | 81 | 14 |
| 4 | 63 | 65 | 2 |
| 5 | 57 | 70 | 13 |
| 6 | 53 | 61 | 8 |
| 7 | 51 | 54 | 3 |
| 8 | 46 | 41 | -5 |
| 9 | 43 | 36 | -7 |
| 10 | 41 | 47 | 6 |
| 11 | 36 | 56 | 20 |
| 12 | 36 | 49 | 13 |
| 13 | 36 | 35 | -1 |
| 14 | 35 | 45 | -10 |
| 15 | 27 | 42 | 15 |
| 16 | 25 | 33 | 8 |
| 17 | 22 | 41 | 19 |
| Total | 797 | 923 | 126[a] |
| Mean | 46.88 | 54.29 | 7.41 |

[a]Used in contingency table.

Table 16

Total Arithmetic Raw Scores of Group B after Using Cuisenaire Rods

| Student | Posttest I | Posttest II | Increase |
|---|---|---|---|
| 1 | 74 | 79 | 5 |
| 2 | 68 | 77 | 9 |
| 3 | 66 | 78 | 12 |
| 4 | 66 | 77 | 11 |
| 5 | 64 | 72 | 8 |
| 6 | 58 | 73 | 15 |
| 7 | 58 | 63 | 5 |
| 8 | 55 | 63 | 8 |
| 9 | 52 | 66 | 14 |
| 10 | 49 | 54 | 5 |
| 11 | 45 | 54 | 9 |
| 12 | 45 | 49 | 4 |
| 13 | 42 | 53 | 11 |
| 14 | 42 | 52 | 10 |
| 15 | 37 | 42 | 5 |
| 16 | 36 | 43 | 7 |
| 17 | 29 | 23 | -6 |
| Total | 886 | 1018 | 132[a] |
| Mean | 52.12 | 59.88 | 7.76 |

[a]Used in contingency table.

Table 17

Conclusions Based on .05
Level of Probability

| Null Hypothesis | Value of Chi-Square | Conclusions |
|---|---|---|
| $H_1$ | .01 | accept |
| $H_2$ | 29.27* | reject |
| $H_3$ | 1.03 | accept |
| $H_4$ | 1.17 | accept |

*Significant at .001 level.

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